INDUCTIVE ACCELERATION OF AN ELECTRICALLY CONDUCTIVE PARTICLE IN A VISCOUS LIQUID

V. M. Korovin

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The characteristics of the motion of a particle in an electrically conducting liquid with constant crossed electric and magnetic fields present have been investigated in connection with the problem of MHD-separation in many papers (for example, see the bibliography in [1]). The separation of electrically conducting particles contained in a dielectric liquid, which can be accomplished with the help of a variable magnetic field [2], is also of practical interest. The ponderomotive force acting on a spherical conducting particle near a straight conductor through which the discharge current of a capacitor bank is flowing is found in this paper, and the motion of a particle in a viscous liquid under the action of this force is investigated.

We shall calculate the ponderomotive force acting on a conducting sphere of radius a located at a distance $l \gg a$ from the axis of a straight cylindrical conductor through which at time $t > 0$ a discharge current $Ie^{-\alpha t}\sin\omega t$ of a capacitor bank of capacitance C, which is included in the circuit in series with an inductance L, charged in advance to a potential difference V, starts to flow. It is assumed that the ohmic resistance of the discharge circuit $R \ll \sqrt{2L/C}$, due to which $\alpha \ll \omega$.

We shall introduce a Cartesian coordinate system Oxyz attached to the center of the sphere, whose Oy axis is antiparallel to the direction of the current during the first half-period and whose Oz axis is directed along the tangent to the circle formed by the intersection of the cylindrical surface coaxial with the conductor which passes through the point O and the plane perpendicular to the Oy axis (Fig. 1).

In the absence of the sphere the magnetic field outside the conductor is represented as follows in complex notation:

$$\mathbf{H}_e = [H_x e^{(i\omega-\alpha)t}, 0, H_z e^{(i\omega-\alpha)t}], \quad i = \sqrt{-1},$$
$$H_x = \frac{iI}{2\pi}\frac{z}{(x+l)^2+z^2}, \quad H_z = -\frac{iI}{2\pi}\frac{x+l}{(x+l)^2+z^2}, \quad I = \frac{V}{\omega L}. \tag{1}$$

Let us switch to the spherical coordinate system r, ϑ, φ with center O in which the polar angle ϑ is figured from the direction of the Oz axis and the azimuthal angle φ is figured from the plane $y = 0$. In this coordinate system the first two terms of the expansion of the field (1) in powers of the ratio $r/l < 1$ are of the form

$$\mathbf{H}_e = (\mathbf{H}_1 + \mathbf{H}_2)\, e^{(i\omega-\alpha)t}, \quad \mathbf{H}_1 = [-H_0\cos\vartheta, H_0\sin\vartheta, 0], \quad H_0 = \frac{iI}{2\pi l},$$
$$\mathbf{H}_2 = \left[H_0\frac{r}{l}\sin 2\vartheta\cos\varphi, H_0\frac{r}{l}\cos 2\vartheta\cos\varphi, -H_0\frac{r}{l}\cos\vartheta\sin\varphi\right]. \tag{2}$$

It is evident that the term H_1 describes a uniform field parallel to the Oz axis.

The magnetic fields inside and outside of the sphere shall be denoted as $h_i = he^{(i\omega-\alpha)t}$ and h_e. In view of the linearity of the electrodynamics equations, one can set

$$\mathbf{h}_e = \mathbf{H}_e + e^{(i\omega-\alpha)t}\nabla\theta,$$

where θ is a function which is harmonic in the exterior of the sphere:

$$\Delta\theta = 0, \quad \Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{r^2\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}. \tag{3}$$

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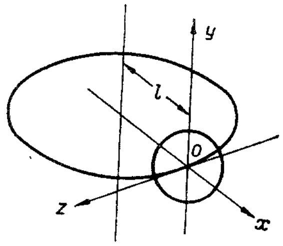

Fig. 1

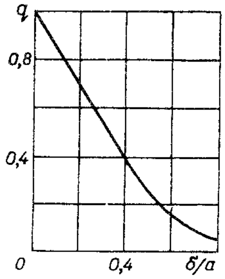

Fig. 2

Substituting h_i into the induction equation and the condition of a solenoid nature for the magnetic field and neglecting small terms of the order of α/ω in comparison with unity, we have

$$\begin{gathered}\Delta h_r - \frac{2}{r^2}\left\{\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}(\sin\vartheta h_\vartheta) + \frac{1}{\sin\vartheta}\frac{\partial h_\varphi}{\partial\varphi} + \left[1 + \frac{i}{2}(\varkappa r)^2\right]h_r\right\} = 0,\\ \Delta h_\vartheta + \frac{2}{r^2}\left\{\frac{\partial h_r}{\partial\vartheta} - \frac{\cos\vartheta}{\sin^2\vartheta}\frac{\partial h_\varphi}{\partial\varphi} - \frac{1}{2}\left[\frac{1}{\sin^2\vartheta} + i(\varkappa r)^2\right]h_\vartheta\right\} = 0,\\ \Delta h_\varphi + \frac{2}{r^2}\left\{\frac{1}{\sin\vartheta}\frac{\partial h_r}{\partial\varphi} + \frac{\cos\vartheta}{\sin^2\vartheta}\frac{\partial h_\vartheta}{\partial\varphi} - \frac{1}{2}\left[\frac{1}{\sin^2\vartheta} + i(\varkappa r)^2\right]h_\varphi\right\} = 0,\\ \frac{\sin\vartheta}{r}\frac{\partial}{\partial r}(r^2 h_r) + \frac{\partial}{\partial\vartheta}(\sin\vartheta h_\vartheta) + \frac{\partial h_\varphi}{\partial\varphi} = 0, \quad \varkappa = \frac{\sqrt{2}}{\delta},\end{gathered} \tag{4}$$

where $\delta = \sqrt{2/\mu_0\sigma\omega}$ is the skin layer thickness, $\mu_0 = 4\pi\cdot10^{-7}$ G/m is the magnetic permeability of a vacuum, and σ is the conductivity of the particle material.

The functions h_r, h_ϑ, and h_φ should be bounded at the center of the sphere, the function ϑ should vanish at infinity, and in addition these functions should provide for continuity of the magnetic field on the sphere surface

$$r = a : \mathbf{H}_1 + \mathbf{H}_2 + \nabla\theta = \mathbf{h}. \tag{5}$$

One of the solutions of the system (4) can be represented in the form [3]

$$h_r = \frac{\partial^2}{\partial r^2}(r\chi) - i\varkappa^2 r\chi, \quad h_\vartheta = \frac{1}{r}\frac{\partial}{\partial\vartheta}(r\chi), \quad h_\varphi = \frac{1}{r\sin\vartheta}\frac{\partial}{\partial\varphi}(r\chi), \tag{6}$$

where χ is a solution of the equation

$$\Delta\chi - i\varkappa^2\chi = 0. \tag{7}$$

Having solved the Laplace equation (3) and the Helmholtz equation (7) with the help of the method of separation of variables, one can show that θ and the auxiliary function χ, in terms of which the solution of the problem (3)-(5) is expressed, are of the form

$$\theta = \frac{1}{r^2}A_1\cos\vartheta + \frac{1}{r^3}A_2\sin 2\vartheta\cos\varphi,$$

$$\chi = \frac{1}{\sqrt{r}}\left[B_1 J_{3/2}(\xi)\cos\vartheta + B_2 J_{5/2}(\xi)\sin 2\vartheta\cos\varphi\right], \quad \xi = \varkappa r\sqrt{-i},$$

where A_k and B_k are unknown constants and $J_{3/2}(\xi)$ and $J_{5/2}(\xi)$ are cylindrical functions of the first kind. Bearing (6) and (5) in mind, we find $h = h_1 + h_2$, where

$$\begin{aligned}&h_{1r} = \frac{2}{r^{3/2}}B_1 J_{3/2}(\xi)\cos\vartheta, && h_{1\vartheta} = \frac{1}{r^{3/2}}B_1\sin\vartheta\left[J_{3/2}(\xi) - \xi J_{1/2}(\xi)\right],\\ &h_{2r} = \frac{6}{r^{3/2}}B_2 J_{5/2}(\xi)\sin 2\vartheta\cos\varphi, && h_{2\vartheta} = \frac{2}{r^{3/2}}B_2\cos 2\vartheta\cos\varphi\left[\xi J_{3/2}(\xi) - 2J_{5/2}(\xi)\right],\\ &h_{1\varphi} = 0, && h_{2\varphi} = \frac{2}{r^{3/2}}B_2\cos\vartheta\sin\varphi\left[2J_{5/2}(\xi) - \xi J_{3/2}(\xi)\right],\\ &A_1 = \frac{a^3 H_0}{2}\frac{J_{5/2}(\xi_0)}{J_{1/2}(\xi_0)}, && A_2 = -\frac{a^5 H_0}{3l}\frac{J_{7/2}(\xi_0)}{J_{3/2}(\xi_0)}, \quad \xi_0 = \varkappa a\sqrt{-i},\\ &B_1 = -\frac{3a^{3/2}}{2\xi_0}\frac{H_0}{J_{1/2}(\xi_0)}, && B_2 = \frac{5a^{5/2}}{6l\xi_0}\frac{H_0}{J_{3/2}(\xi_0)}.\end{aligned} \tag{8}$$

Knowing $\mathbf{h}$, it is not difficult to find the distribution of the Foucault currents $\mathbf{j}$ inside the sphere:

$$\mathbf{j} = (\mathbf{j}_1 + \mathbf{j}_2)e^{(i\omega-\alpha)t}, \quad \mathbf{j}_k = \operatorname{rot} \mathbf{h}_k, \quad k = 1, 2$$

or in the projections

$$j_{1r} = j_{1\vartheta} = j_{2r} = 0, \quad j_{1\varphi} = -\frac{i\varkappa^2}{\sqrt{r}} B_1 J_{3/2}(\xi) \sin\vartheta,$$
$$j_{2\vartheta} = \frac{2i\varkappa^2}{\sqrt{r}} B_2 J_{5/2}(\xi) \cos\vartheta \sin\varphi, \quad j_{2\varphi} = \frac{2i\varkappa^2}{\sqrt{r}} B_2 J_{5/2}(\xi) \cos 2\vartheta \cos\varphi. \tag{9}$$

Next, neglecting small terms of the order of α/ω and $(a/l)^2$ in comparison with unity, we calculate with the help of (8) and (9) the density of the ponderomotive force f averaged over the period of the current:

$$f_r = -\frac{\mu_0}{2} e^{-2\alpha t} \operatorname{Re}\left[j_{1\varphi}^*(h_{1\vartheta} + h_{2\vartheta}) + j_{2\varphi}^* h_{1\vartheta}\right],$$
$$f_\vartheta = \frac{\mu_0}{2} e^{-2\alpha t} \operatorname{Re}\left[j_{1\varphi}^*(h_{1r} + h_{2r}) + j_{2\varphi}^* h_{1r}\right], \quad f_\varphi = -\frac{\mu_0}{2} e^{-2\alpha t} \operatorname{Re}\left[j_{2\vartheta}^* h_{1r}\right],$$

where $\mathbf{j}_k^* = (j_{kr}^*, j_{k\vartheta}^*, j_{k\varphi}^*)$ is a vector which is complex-conjugate to $\mathbf{j}_k$. Switching to the Cartesian coordinate system and integrating f over the volume of the sphere, we find the total ponderomotive force F which is acting on the sphere:

$$F_x = \frac{\mu_0 q}{4\pi}\left(\frac{a}{l}\right)^3 I^2 e^{-2\alpha t}, \quad F_y = F_z = 0,$$
$$q = \frac{0.5(\beta_1^2 + \beta_2^2) - \zeta^{-1}(3\beta_1\gamma_2 + 2\beta_2\gamma_1) + 2\zeta^{-2}(3\beta_1\beta_2 + \gamma_1^2) - 6\zeta^{-3}\beta_1\gamma_1}{\gamma_1(\gamma_2 - 2\zeta^{-1}\beta_2 + 2\zeta^{-2}\gamma_1)}, \tag{10}$$
$$\beta_{1,2} = \operatorname{sh} 2\zeta \mp \sin 2\zeta, \quad \gamma_{1,2} = \operatorname{ch} 2\zeta \mp \cos 2\zeta, \quad \zeta = \frac{2a}{\delta}.$$

In the approximation under discussion the principal moment of the ponderomotive forces is equal to zero. The force F_∞ acting on an ideally conducting sphere is easily calculated from formula (10):

$$F_{\infty x} = \lim_{\zeta\to\infty} F_x = \frac{\mu_0}{4\pi}\left(\frac{a}{l}\right)^3 I^2 e^{-2\alpha t}, \quad F_{\infty y} = F_{\infty z} = 0.$$

The plot given in Fig. 2 indicates a strong dependence of F_x on the relative skin layer thickness δ/a. In the case of a thick skin layer the leading term of the expansion of F_x in powers of $(a/\delta)^4 < 1$ is of the form

$$F_x \simeq \frac{2\mu_0}{315\pi}\left(\frac{a}{l}\right)^3\left(\frac{a}{\delta}\right)^4 I^2 e^{-2\alpha t}.$$

Making use of formula (10), we shall consider the effect of a variable magnetic field on the gravitational settling of a single conducting particle in a quiescent liquid near a vertical conductor through which at $t > 0$ a discharge current flows. The horizontal motion of the particle caused by the ponderomotive force is found from the solution of the problem

$$\frac{4}{3}\pi a^3 \rho_p \frac{d^2 l}{dt^2} = -6\pi\rho\nu a \frac{dl}{dt} - \frac{2}{3}\pi a^3 \rho \frac{d^2 l}{dt^2} - 6\rho a^2 \sqrt{\pi\nu} \int_0^t \frac{d^2 l(\tau)}{d\tau^2} \times$$
$$\times \frac{d\tau}{\sqrt{t-\tau}} + \frac{\mu_0 q}{4\pi}\left(\frac{a}{l}\right)^3 I^2 e^{-2\alpha t}, \tag{11}$$
$$t = 0:\ l = l_0, \quad \frac{dl}{dt} = 0,$$

where ρ and ν are the density and kinematic viscosity of the liquid and ρ_p is the density of the particle material. The terms which appear on the right-hand side of Eq. (11) describe the Stokesian drag force, the effect of additional masses, the Basse force, and the ponderomotive force. Neglecting the variation of F_x associated with the displacement of the particle during the discharge of the capacitor bank, we perform a Laplace transformation in (11). With $\rho \neq 1.6\rho_p$ the operator solution of the problem (11) $X(s) \risingdotseq l(t)$ can be represented as follows:

$$X(s) = \frac{l_0}{s} + \frac{\lambda_3}{\lambda_1 - \lambda_2}\frac{1}{s(s+2\alpha)}\left(\frac{1}{\sqrt{s} - \lambda_1} - \frac{1}{\sqrt{s} - \lambda_2}\right), \tag{12}$$

$$\lambda_{1,2} = -b_1 \pm \sqrt{b_1^2 - b_2}, \quad \lambda_3 = \frac{3q\mu_0}{8\pi^2\rho_1 l_0^3} I^2,$$

$$b_1 = \frac{9\rho\sqrt{\nu}}{2a\rho_1}, \quad b_2 = \frac{9\rho\nu}{a^2\rho_1}, \quad \rho_1 = 2\rho_p + \rho.$$

The originals of each of the factors appearing in (12) occur in the table given in [4]. Applying the multiplication theorem and using formula 3.383.1 from [5], one can find the original of the function X(s):

$$l(t) = l_0 + \frac{\lambda_3}{2\alpha(\lambda_1 - \lambda_2)} [\eta(t, \alpha, \lambda_1) - \eta(t, \alpha, \lambda_2)),$$

$$\eta(t, \alpha, \lambda_k) = \frac{1}{\lambda_k^2 + 2\alpha} \left\{ \lambda_k e^{-2\alpha t} \left[1\ 2\ \ \lambda_k \sqrt{\frac{t}{\pi}} \Phi\left(\frac{1}{2}, \frac{3}{2}; 2\alpha t\right) \right] + \right. \tag{13}$$

$$\left. + \frac{2\alpha}{\lambda_k} e^{\lambda_k^2 t} \left[1 - \frac{2}{\sqrt{\pi}} \int_0^{-\lambda_k \sqrt{t}} e^{-u^2} du \right] \right\} - \frac{1}{\lambda_k}, \quad k = 1, 2,$$

where $\Phi(1/2, 3/2; 2\alpha t)$ is the degenerate hydrogeometric function. The limiting form of the law of horizontal motion of the particle (13) at $\sqrt{t} \gg \max(|\lambda_1|^{-1}, \alpha^{-1/2})$

$$l - l_0 = \frac{\mu_0 q}{48\alpha\rho\nu l_0^3} \left(\frac{a}{\pi} I\right)^2 \left[1 - \frac{a}{\sqrt{\pi\nu t}}\right],$$

obtained with the help of asymptotic expansions of the error integral and the degenerate hypergeometric function at large values of their arguments [6], permits estimating the maximum displacement of a particle under the action of a variable magnetic field generated by a discharge current.

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